Harmonics and Inverters

Course No: E04-050
Credit: 4 PDH

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Introduction

Static UPS are almost perfect electric generators. They have high reliability and, by nature, ensure (within the battery operation limits) the uninterrupted power supply. Regarding electrical features, the inverter (which is part of the UPS generator) possesses characteristics superior to those of the mains, in terms of frequency and voltage stability. The only doubtful feature is its ability to provide a sinusoidal voltage regardless of the shape of the current drawn by the load. The objective of this course is to elaborate on this topic and to show that modern inverters are excellent generators of sinusoidal voltage even when they supply non-linear loads. This is considered as normal since UPS devices are designed and very often utilised to supply computer/ microprocessor systems which draw non-sinusoidal currents.

Harmonic assessment of a periodic function

Since non-sinusoidal AC currents and voltages are the main focus of this course, it will be useful to review the electric quantities in the presence of harmonics. Fourier analysis claims that any non-sinusoidal periodic function can be expressed by a series of terms that consist of:

- A sinusoidal term at fundamental frequency;
- Sinusoidal terms whose frequencies are whole multiples of the fundamental frequency (harmonics); and
- A continuous component (DC component).

The equation that presents the harmonic analysis of a periodic function is:

$$y(t) = Y_0 + \sum_{n=1}^{n=\infty} Y_n \sqrt{2} \sin(n\omega t - \varphi_n)$$

Where:

- Y_0: value of continuous component typically equal to zero and considered as such hereafter;
- Y_n: effective value of harmonic of order n;
- ω: fundamental frequency pulsation; and

- φ_n: harmonic component at t=0, phase shift angle.

Effective value of non-sinusoidal alternating values

Application of the general equation:

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y^2(t)dt}$$

Gives harmonic representation:

$$Y_{rms} = \sqrt{\sum_{n=1}^{n=\infty} Y_n^2}$$

Harmonic distortion rates

This indicator known as harmonic distortion or distortion factor represents the ratio of the harmonics effective value (n ≥ 2) to alternating quantity:

$$THD\ \% = DF\ \% = 100\ \frac{\sqrt{\sum_{n=2}^{n=\infty} Y_n^2}}{\sqrt{\sum_{n=1}^{n=\infty} Y_n^2}}$$

Global distortion rate

This indicator shows the ratio of the harmonics effective value to fundamental:

$$D\% = 100\frac{\sqrt{\sum_{n=2}^{n=\infty} Y_n^2}}{Y_1}$$

When distortion rate is low, which is typically the case for the voltage, the two definitions lead to the same result. For Instance, if:

$$\sqrt{\sum_{n=2}^{n=\infty} Y_n^2} = 10\%\ of\ Y_1$$

The first equation gives:

$$THD = DF = 100\frac{\sqrt{(0.1)^2}}{\sqrt{1+(0.1)^2}} = 9.95\%$$

While the second equation gives:

$$D\% = 100\frac{0.1}{1} = 10\%$$

We will mainly use global distortion rate that corresponds to a more analytical view of the influence of harmonics on a non-deformed wave.

Individual harmonic rate

This indicator represents the ratio of the n^{th} harmonic order effective value to the alternating quantity or to the fundamental alone.

- This first representation is given with the equation:

$$H_n\% = 100\frac{Y_n}{\sqrt{\sum_{n=1}^{n=\infty} Y_n^2}}$$

- The second representation is given with the equation:

$$H_n\% = 100\frac{Y_n}{Y_1}$$

The second definition will be used in the rest of the course.

Power factor and $\cos\varphi_1$

The power factor is the ratio of the effective power, P, to the apparent power, S, as shown with the following equation:

$$\lambda = \frac{P}{S}$$

This power factor should not be confused with the phase shift angle factor ($\cos\varphi_1$) which represents the cosine of angle formed by the phase elements of voltage and current fundamental components:

$$\lambda_1 = \cos\varphi_1 = \frac{P_1}{S_1}$$

Where:

- P_1: effective power of fundamental component; and
- S_1: apparent power of fundamental component.

Distortion factor ϑ

Distortion factor defines the relation between power factor, λ, and $\cos\varphi_1$:

$$\vartheta = \frac{\lambda}{\cos\varphi_1}$$

When voltages and currents are perfectly sinusoidal the distortion factor is equal to 1 and $\cos\varphi_1$ is equal to the power factor.

Crest factor

Crest factor is the ratio of crest value to the effective value of a periodic signal.

Relationship between current and voltage distortions

For a particular voltage source, it is always possible to determine output impedance, even if it is frequency dependent. It is possible to calculate the corresponding voltage harmonic for each current harmonic, including situations when this impedance is independent of the current value (linear case). Therefore, it is possible to deduce the individual harmonic rate (percentage). The effective value of the n^{th} voltage harmonic equals:

$$U_n = Z_{sn} I_n$$

Where:

- Z_{sn}: output impedance for harmonic n; and

- I_n: effective current of harmonic n.

The individual rate of n^{th} harmonics for this voltage equals:

$$H_n = \frac{U_n}{U_1}$$

Where:

- U_1: fundamental voltage effective value.

Therefore, the voltage global distortion rate is calculated using the expression:

$$D\ \% = 100\ \frac{\sqrt{\sum_{n=2}^{n=\infty} U_n^2}}{U_1}$$

And also:

$$D\ \% = 100\sqrt{\sum_{n=2}^{n=\infty} H_n^2}$$

Therefore, the input impedance for different harmonic frequencies plays an important role in bringing about the onset of voltage distortion. The higher the input impedance, the higher the voltage distortion rate will be for a given non-sinusoidal current.

Conventional sources' impedances

Commonly, the generator impedance, Z_s, (at 60 Hz) is given as a percentage of the load nominal impedance, Z_c:

$$Z_s\ \% = 100\frac{Z_s}{Z_C}$$

Hence, for the nominal current, the voltage drop across this impedance represents the value of this source impedance:

$$\frac{Z_s I_n}{U_n}\% = 100\frac{Z_s I_n}{U_n}$$

Where:

$$Z_c I_n = U_n$$

$$\frac{Z_s I_n}{U_n}\% = 100\frac{Z_s I_n}{Z_c I_n} = 100\frac{Z_s}{Z_c}$$

Transformer impedance

Figure 1 shows an equivalent circuit diagram of a single phase transformer seen from the secondary side. The transformer impedance consists of an inductance, L, connected in series with a resistance, R. Relative impedance equivalent value is expressed by the transformer short-circuit voltage, U_{cc}.

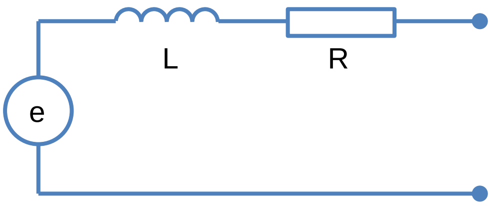


Figure 1. Equivalent circuit diagram of a single phase transformer seen from the secondary side

The short-circuit voltage is the voltage that must be applied across a winding in order to induce a nominal current in the other winding which is also short-circuited:

$$U_{cc}\% = 100\frac{U_{cc}}{U_n}$$

$$U_{cc}\% = 100\frac{Z_s I_n}{U_n} = 100\frac{Z_s}{Z_c} = Z_s\%$$

Short-circuit voltage is made up of two parts, U_{ccR} and U_{ccx}, as shown in Figure 2.

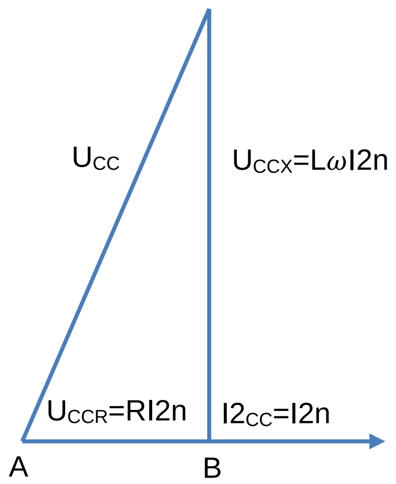


Figure 2. Vector diagram of a transformer (values referred to secondary side)

In the case of distribution or general-purpose transformers, the value of U_{ccx} is in the 4 - 6 % range, whereas the value of U_{ccR} is of the order of 1 % to several %. U_{ccR} becomes correspondingly smaller as the power rating of the transformer increases. With respect to harmonics, since only the inductance impedance is frequency dependent, it is the inductance alone which affects the transformer performance.

In the case of three phase transformers, it is mandatory to consider the different connection types of primary and secondary windings, as these exert impact on the source impedance for some harmonics (especially, third harmonic and multiples of 3). Actually, in the case of a transformer which supplies to each of its secondary windings distorted and balanced currents comprising harmonics of order 3 and multiples of 3, say 3 k, and considering that these currents are balanced, it is possible to write for each of these phases:

$$I_{13k} = I \sin 3k\omega t$$

$$I_{23k} = I \sin 3k \left(\omega t - \frac{2\pi}{3}\right)$$

$$I_{33k} = I \sin 3k \left(\omega t - \frac{4\pi}{3}\right)$$

Or

$$I_{13k} = I \sin 3k\omega t$$

$$I_{23k} = I \sin(3k\omega t - k2\pi)$$

$$I_{33k} = I \sin(3k\omega t - k4\pi)$$

Equations show that the three currents are in phase. This leads to the presence of much higher currents than originally expected in the neutral conductor of some wiring installations.

Hence, the behaviour of a transformer towards these harmonics depends on the transformer homopolar impedance, Z_h. Two types of secondary windings are appropriate for not amplifying or decreasing harmonic distortions.

When primary windings are delta- or star-connected with the neutral point connected to the source neutral as shown in Figure 3, the harmonic impedances of 3rd order are neither encouraged nor discouraged (Z_h=Z_d). The transformer behaves as three single phase transformer units.

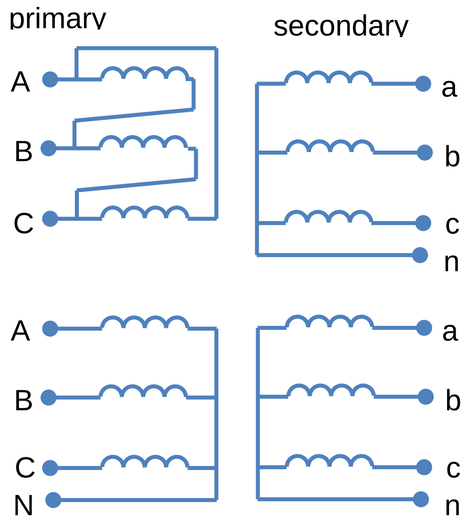


Figure 3. Three phase transformers winding connections which have a homopolar impedance, Zh, equal to a direct impedance, Zd

ZIGZAG connected secondary side - ZIGZAG connections provide minimum distortion in secondary winding. Actually, in this case, the harmonic currents of 3k order do not flow in the transformer primary, and the impedance, Z_s, does not depend on the secondary windings. Hence, the inductance is very low: U_{ccx}≈1%, and the resistance is decreased by approximately half in comparison with the resistance of delta star connected transformer of the same rating. Figure 4 and the following calculation show why currents of pulsating frequency 3kω are not found in the transformer primary (homopolar current is zero).

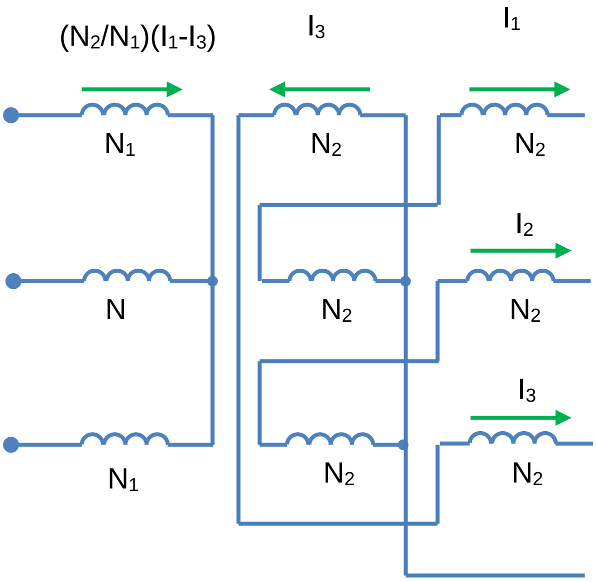


Figure 4. Transformer with ZIGZAG connected secondary and attenuation of 3k order harmonics

$$For\ a\ turn\ ratio\ \frac{N_2}{N_1}$$

the current flowing in the primary winding 1 is:

$$\frac{N_2}{N_1}(i_1 - i_3)$$

With

$$i_1 = I_{13k} = I \sin 3k\omega t$$

$$i_3 = I_{33k} = I \sin 3k\left(\omega t - \frac{4\pi}{3}\right) = I \sin(3k\omega t - 4\pi)$$

This gives:

$$\frac{N_2}{N_1}(i_1 - i_3) = 0$$

Hence, ZIGZAG connected secondary winding acts as an attenuator to 3k order harmonics. This transformer type is typically used as an output transformer for classic high rating inverters.

Generally, the other connection types are to be avoided, especially those that do not allow the neutral to be distributed in the secondary.

Alternator Impedance

An alternator can be modelled by a voltage source in series with an inductance and a resistance. Nevertheless, this inductance has different values which depend on the speed of current changes to which it is related. During current change, the equivalent reactance progressively passes from a value called subtransient to its synchronous value via a transient value. Different values reflect the change of the alternator magnetic flux. Regarding current harmonics, only the sub-transient reactance needs to be considered. This reactance, also known as "longitudinal sub-transient reactance", is labelled as X''_d. For an alternator of current production, this reactance equals 15 - 20 %. In traditional machines with optimised design, a value of 12 % can be achieved. Some manufacturers claim values of 6 %, for certain, special machines.

The alternator output impedance is considerably higher than the transformer impedance, except in very special cases. Therefore, the same applies to the voltage distortion rate in the presence of distorted currents.

Inverter output impedance

Inverter impedance depends on the output impedance of its filter and the type of used regulation.

Inverter principle

An inverter is composed of a converter known as a "mutator" e.g. switching device which converts the DC voltage provided by a rectifier or a DC battery into AC voltage. In a single phase unit, there are two ways to perform this conversion:

- Half-bridge converter (Figure 5a); or

- Full-bridge converter (Figure 5b).

Figure 5. (a) Figure 5. (b)

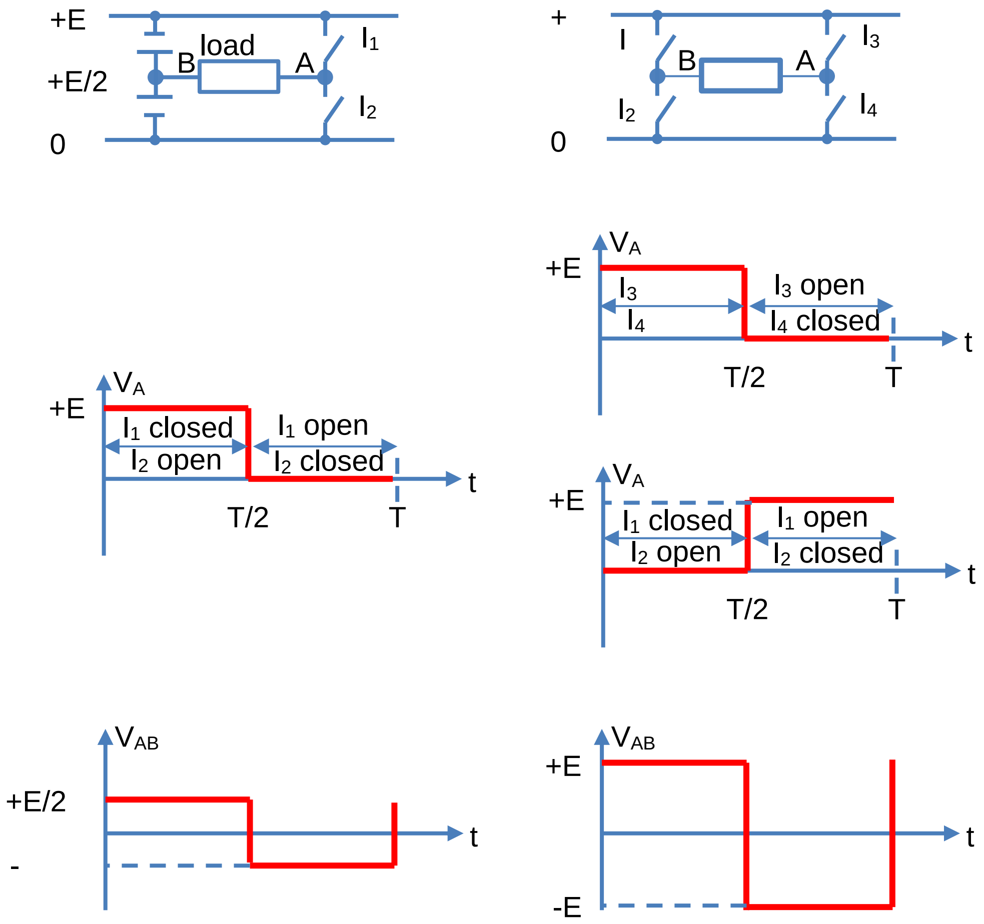


Figure 5. (a). Principle of switching device (mutator) half-bridge converter (b). Principle of mutator full-bridge converter

The square wave voltage that appears between A and B is then filtered to obtain a sinusoidal voltage wave with a low distortion rate in the output of the unit. In reality, the switching device (mutator) generates several positive and negative pulses (as shown in Figure 6) which makes it possible to decrease the filter size and to have a faster acting voltage regulator.

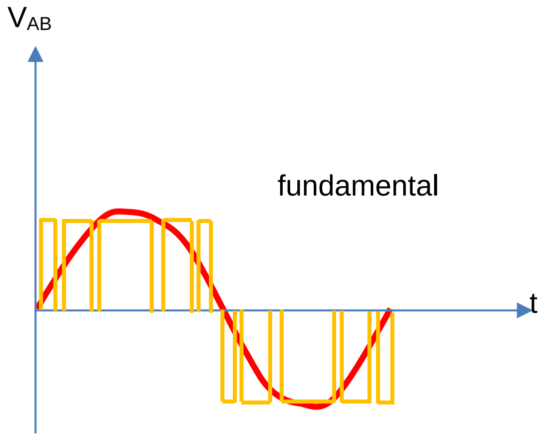


Figure 6. Switching unit (mutator) output voltage with 5 pulses per half period

By modulating the relative time intervals corresponding to conduction and non-conduction periods, it is possible to "spread" the voltage during the period in such a way to make the conduction time of the switching device proportional to the instantaneous value of the fundamental.

This method is known as Pulse Width Modulation (PWM). The filter installed behind the switching element (mutator) is typically of the L and C type (as shown in Figure 7). Hence, the inverter is a voltage source with the filter impedance connected in series.

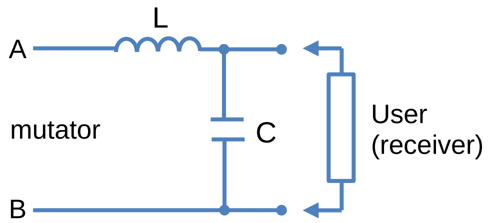


Figure 7. Inverter output filter

Voltage, V, is the voltage measured at zero load, and the impedance consisting of L and C elements connected in parallel is the impedance measured when terminals A and B are short-circuited (as shown in Figure 8).

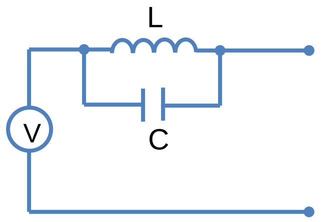


Figure 8. Inverter equivalent circuit diagram seen from its output

Classic inverters

In situations when the commutation frequency is low, regulation can:

- deal with current changes drawn by user equipment;

- compensate for DC battery (or rectifier) voltage change; and

- experience problems in dealing permanently with current changes due to harmonics created during half cycle. In these inverters, the output impedance is equal to the filter impedance. Hence, they can be described as classic inverters since operationally they work in the same way as the early design devices (due to semi-conductors limited capacity to operate at high frequencies). Hence, the output impedance of these inverters is frequency-dependent. It can be presented with diagram as shown in Figure 9.

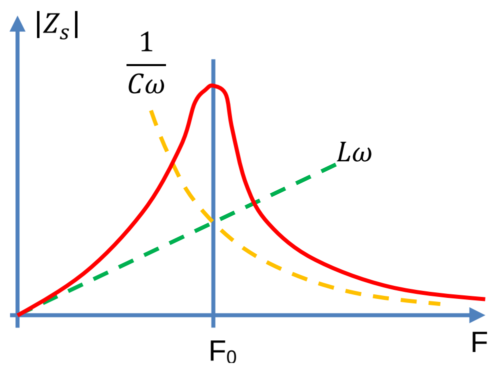


Figure 9. Change of output impedance in a classic frequency inverter

- At low frequencies, filter impedance is nearly equal to Lω

- At high frequencies, filter impedance differs little from $\frac{1}{C\omega}$
- At resonant frequency, $F_0 = \frac{1}{2\pi\sqrt{LC}}$

Filter impedance assumes a high value that can attain, in terms of magnitude, the value of the equipment's nominal load impedance (Z_s=100% Z_c).

Hence, F_0 is selected such that it does not correspond to the possible current harmonic frequency, i.e. 210 Hz (harmonic currents of order 4 are non-existent or are very small in amplitude). Therefore, different methods have been devised by manufacturers in an effort to decrease the output impedance. For example:

- Additional filters; and
- Specific connection circuits for the transformer inserted behind the three-phase switching element (mutator).

Inverters with PWM and regulation

When the switching unit's (mutator) switching frequency is high (at least several kHz) and the regulation system allows quick intervention through the modification to pulse widths during the same period, it is possible to keep the inverter output voltage within its distortion limits even when dealing with highly distorted currents.

The block diagram of such inverter is shown in Figure 10. It can be seen that the output voltage, V_s, is constantly compared with a reference voltage, U_{ref}, which is sinusoidal and has a low distortion rate (<1%). The voltage difference ε is processed by a correction circuit of transfer function, C(p), whose goal it is to ensure the performances and the stability of control circuit systems. The voltage issued from this correction circuit is then amplified by the switching unit (mutator) and its ancillary control circuit with an amplification gain, A.

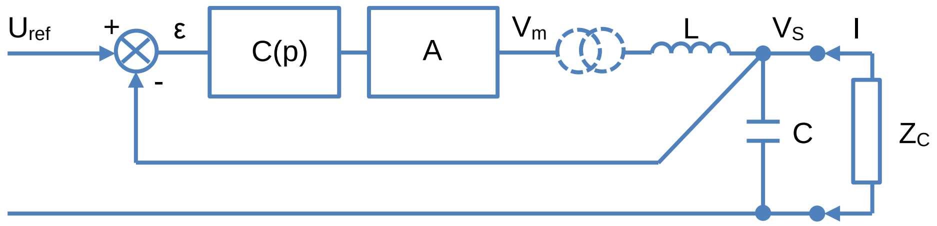


Figure 10. PWM inverter block diagram

The voltage, V_m, provided by the switching element is shaped by the filter that consists of L and C elements before becoming the output voltage, V_s. The following needs to be considered:

- Transformer impedance, in order to get the total value of inductance. (Typically, the inductance is integrated within the transformer and does not appear in circuit diagrams for that reason); and

- Switching unit's output impedance is not necessarily negligible.

Generally, it is useful to present the whole output circuit part (switching unit + filter) in the form of a series impedance, Z_1, together with a parallel impedance, Z_2, as shown in Figure 11.

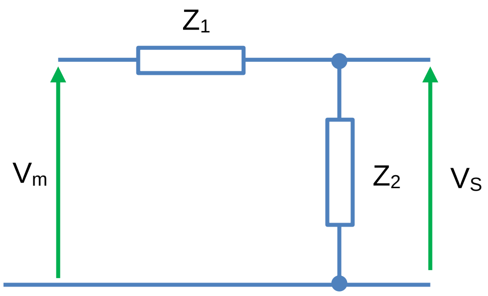


Figure 11. Switching unit's equivalent circuit diagram seen from output

Using the Thevenin theorem, it is possible to transform the circuit diagram into the one in Figure 12. V'_m is the voltage measured at no load. Therefore:

$$V'_m = V_m \frac{Z_2}{Z_1 + Z_2}$$

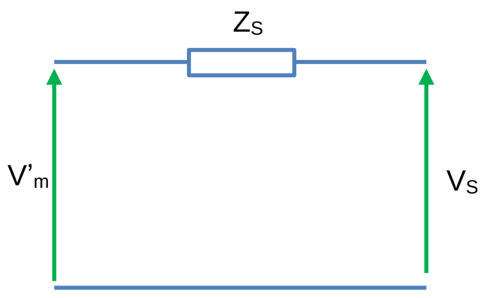


Figure 12. Switching unit's transformed equivalent circuit diagram seen from output

Therefore:

$$Z_s = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Ratio $\frac{Z_2}{Z_1+Z_2}$ is the filter transfer function, H(p), shown as:

$$H(p) = \frac{Z_2}{Z_1 + Z_2}$$

Also, it is convenient to replace the product C(p) x A by μ(p) which represents the transfer function of action chain. In that case, the block diagram is shown in Figure 13. Where Z_s is the output impedance in the absence of regulation, as is the case of classic inverters.

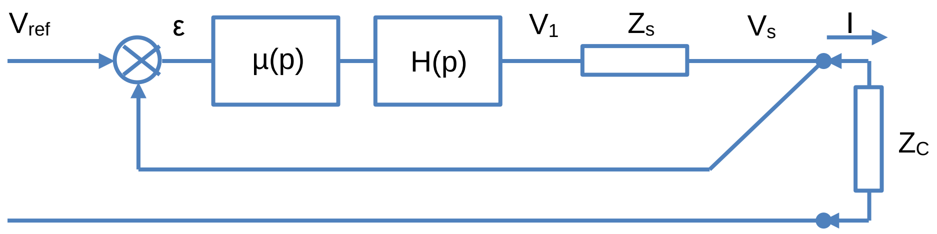


Figure 13. PWM inverter transformed block diagram

When a current is taken by the load, a voltage drop appears at the terminals of the output impedance Z_s, such that:

$V_1 - V_s = Z_s I$

Next:

V1 =ε μ(p) H(p)

$\varepsilon = V_{ref} - V_s$

$V_1 = (V_{ref} - V_s)\ \mu(p)\ H(p)$

$V_1 = V_s + Z_{sI}$

$V_s + Z_{sI} = (V_{ref} - V_s)\ \mu(p)\ H(p)$

Hence:

$V_s\ [1 + \mu(p)\ H(p)] = V_{ref}\ \mu(p)\ H(p) - Z_{sI}$

Therefore:

$$V_s = V_{ref} \frac{\mu(p) \cdot H(p)}{1 + \mu(p) \cdot H(p)} - \frac{Z_{sl}}{1 + \mu(p) \cdot H(p)}$$

The first expression represents the result obtained for a conventional control system with no disturbance. Here, the disturbance is introduced by means of current, I, flowing through the internal impedance, Z_s. In the absence of regulation, the term denoting the disturbance would have assumed a value of Z_{sI}. With regulation, this disturbance is limited to:

$$\frac{Z_{sl}}{1 + \mu(p) \cdot H(p)}$$

Everything happens as if the inverter's output impedance was divided by:

$1 + \mu(p)\ H(p)$

Also, it is convenient to complete additional calculations. In the band-pass of regulation, the product $\mu(p)\ H(p)$ being ≥ 1, calculations are as follows:

$1 + \mu(p)\ H(p) \approx \mu(p)\ H(p)$

$$Z_s' \approx \frac{Z_s}{\mu(p) \cdot H(p)}$$

Since:

$$Z_s = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

And

$$H(p) = \frac{Z_2}{Z_1 + Z_2}$$

Therefore:

$$Z_s' \approx \frac{Z_1 Z_2}{Z_1 + Z_2} \cdot \frac{1}{\mu(p)} \cdot \frac{Z_1 + Z_2}{Z_2}$$

Hence:

$$Z_s' \approx \frac{Z_1}{\mu(p)} \cdot$$

This implies that in the band-pass of regulation, the inverter's output impedance is equal to the filter's series impedance for the whole output circuit divided by the amplification gain of the action chain. Beyond the regulation's band-pass, the output impedance becomes again the impedance of filter which by then becomes the impedance of a capacitor providing a low impedance at high frequencies. Therefore, output impedance's curve shape as a function of frequency is shown in Figure 14.

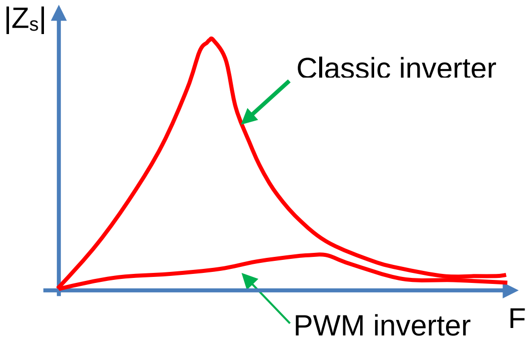


Figure 14. Cross-comparison of output impedances between classic inverter and PWM inverter in function of frequency

With PWM inverters, the output impedance stays very low up to high frequencies and the output voltage distortion due to circulating currents, even highly distorted currents, can be neglected.

Current limitation

The semi-conductors used in switching units can provide a maximum current, above which their performance can no longer be guaranteed. Hence, it is suggested to limit the current to this maximum value in order to ensure service reliability. As soon as the current taken by the load surpasses the maximum value set for the inverter, the latter becomes a generator of constant current until the current value required by the load drops below the fixed threshold limit. In this situation, the output voltage does not follow the shape of the reference voltage and stays distorted as long as the load current surpasses the threshold limit.

Such situations are encountered when single-phase loads consisting of a capacitor in front of a rectifier give a high crest factor. Typically, the latter is of the order of 3 (crest value ≈ 3 times the current effective value) whereas for a pure sine wave it is 2.

Line impedance

Generally, the line impedance can be modelled as an inductance, L, connected in series with a resistance, R, as shown in Figure 15.

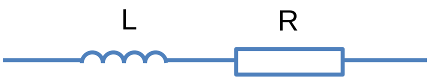


Figure 15. Line equivalent circuit

Inductance value hardly depends on the conductor section and is typically around 0.1 Ω/km which is roughly equivalent to 0.3 µH/m. Resistance value depends on the cable section and is taken as R = 20 Ω/km for 1 mm2 section, e.g., a cable of 16 mm2 section exhibits a resistance of 1.25 Ω/km and a reactance of only 0.1 Ω/km. As a rough approximation, it will be possible to model a cable only by its resistance in the case of small and medium size power rating installations. In those installations the use of small section conductors is quite typical.

For harmonic frequencies, it might be mandatory to consider the skin effect. The equivalent conduction thickness in a copper conductor, known as skin thickness, is given by the equation:

$$a(mm) = \frac{66}{\sqrt{F_{(Hz)}}}$$

Therefore, at 50 Hz, the skin thickness is 9.3 mm, whereas at 1 kHz, it is decreased to 2.1 mm. Hence, the skin effect must be considered for large section conductors, which typically conduct high order harmonic currents.

Line impedance impact on voltage distortion

Since the line impedance is an addition to the source impedance, it increases the voltage distortion rate in installations drawing distorted currents. Figure 16 presents a situation where installation, U_2, takes a highly distorted current.

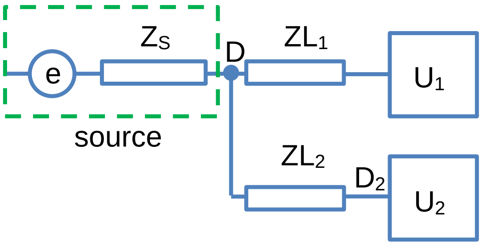


Figure 16. Supply to polluter receiver (U2) through special line

When this happens, the distortion rate measured at its input terminals is D_2. Nevertheless, since the impedance divider consists of Z_s and Z_{L2}, a distortion rate, D, measured at the output terminals of the source D is smaller than D_2. Therefore, to minimise the impact of receiver installations, which produce harmonic currents in other consumers (loads), it is suggested to supply the receiver installations through a special line. Figure 17 presents the variation of output impedances for different sources with the same power rating, in function of frequency.

It is clear that the PWM inverter shows by far the lowest output impedance. For clarity, Figure 18 shows three sources, each with the same impedance at 150 Hz. Hence, it is apparent that the impedance of a classic transformer as well as the impedance of the supply line must both be considered when distorted currents are to be supplied to a load.

The PWM inverter is by far the best generator in terms of its ability to minimise the voltage harmonic distortion. It is 5 to 6 times better than a transformer of the same rating.

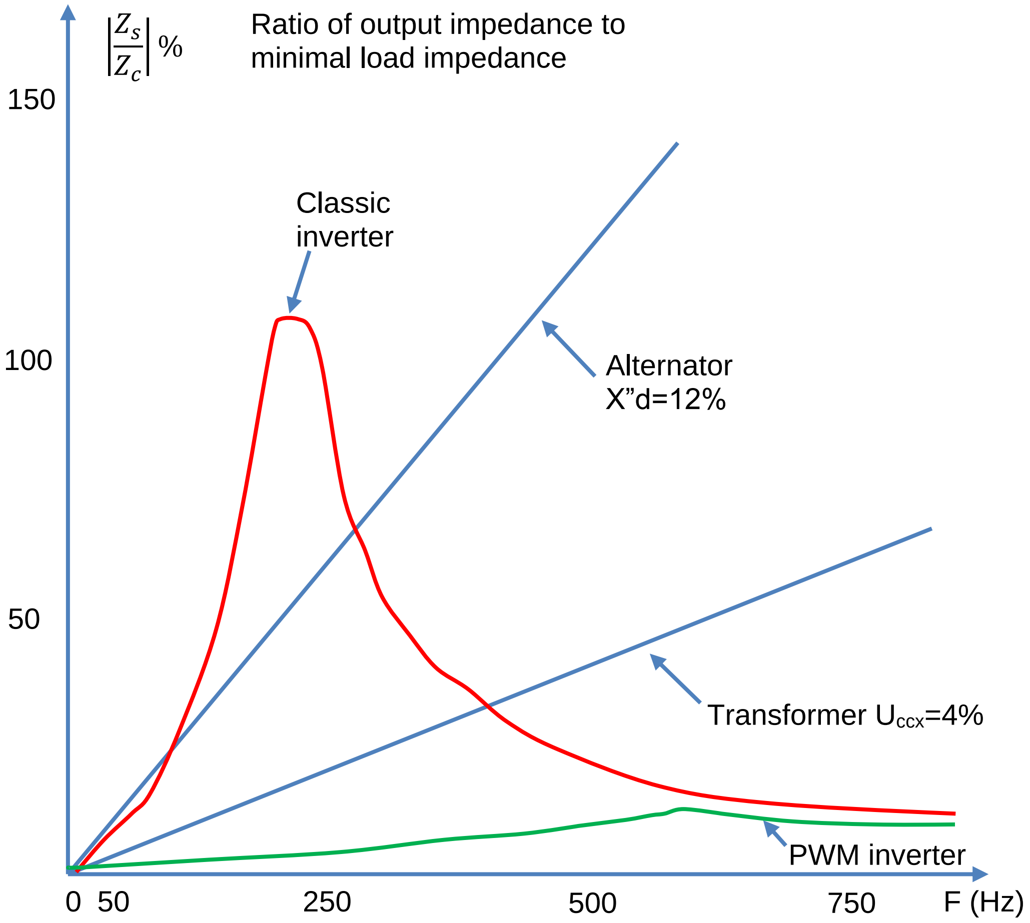


Figure 17. Output impedance of various sources in function of frequency

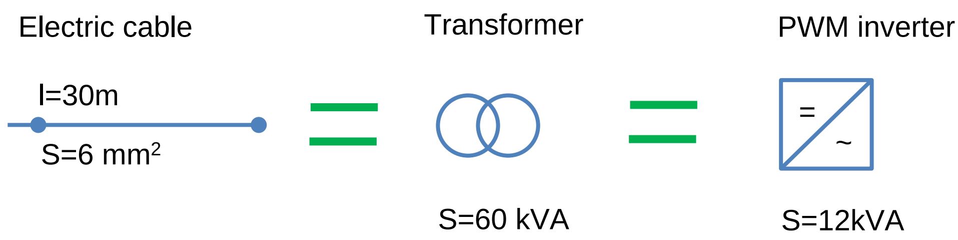


Figure 18. Sources with same impedance at 150 Hz

Micro and mini-computer loads

These single-phase loads, as many other types of electronic devices, are supplied with switched-mode power supplies. Therefore, a load of RCD type (resistances, capacitors, diodes) characterises inverters of rating below 3 kVA. RCD type loads consist of a Graetz full-bridge converter which is preceded by a capacitor. The capacitor serves as an energy storage reservoir in order to provide current to the load between two successive peaks of the rectified voltage. The supply source is modelled by voltage, *e*, and output impedance, Z_s. The capacitor's discharge time constant is fixed at 125 ms for the examples in this chapter (as shown in Figure 19). Current, *i*, starts to flow when voltage, *e*, surpasses the DC voltage, U, and flows for a relatively short time to recharge the capacitor to its nominal voltage.

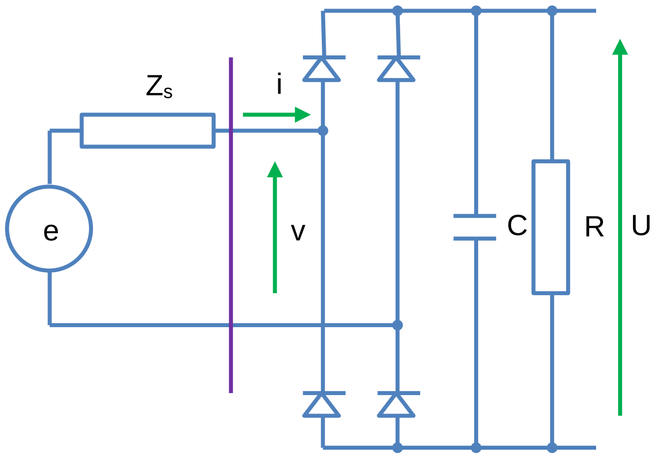


Figure 19. Basic configuration of micro and mini-computer type load

Figure 20 presents the voltages and currents obtained with a relatively low source impedance that consist of an inductance and a resistance such that their short-circuit voltages referred to the load power are respectively U_{ccx}=2 % and U_{ccR}=2 %. The distortion rate of the voltage, *v*, in the rectifier input is important since it reaches 7.5 % despite low source impedance.

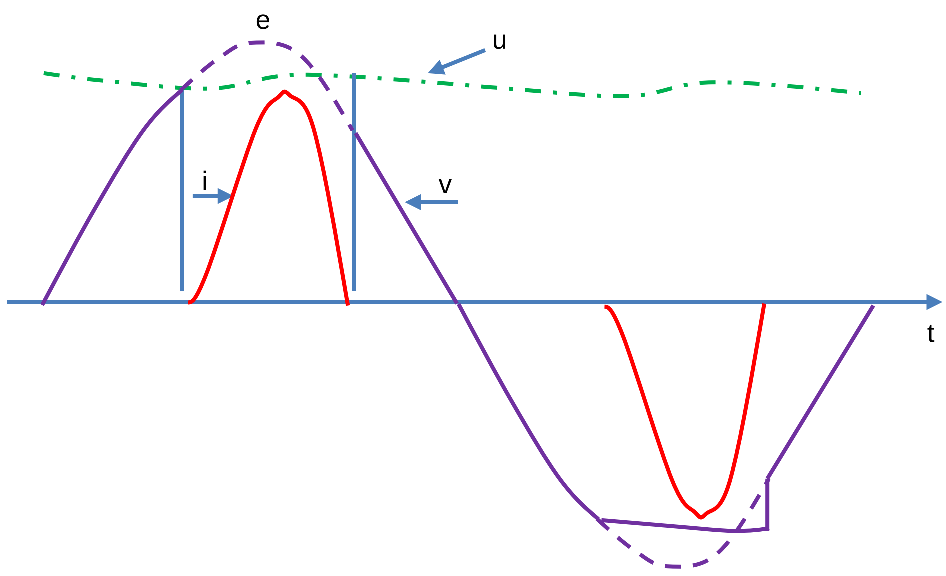


Figure 20. 1 kW computer type load currents and voltages such that: UCCX=2% and UCCR=2%

The current, *i*, starts to flow as soon as the voltage, *e*, becomes higher than *U* but its rate of rise is limited by the source inductance. This inductance extends the time of current circulation when voltage, *e*, becomes smaller than *v*. Hence, the value of the source inductance determines the shape of current, *i*. Current is highly distorted in comparison with a perfect sine wave and is slightly out of phase with respect to the source voltage.

Source impedance influence

In the previous example it was demonstrated that the load cannot be considered as a generator of harmonic current, but on the contrary, that the current highly depends on the source impedance. Figure 21 presents the change of current, *i*, and voltage, *v*, in the rectifier input, when the source impedance varies from U_{ccx} = 0.25 % to U_{ccx} = 8 % whilst the resistive part remains constant at U_{ccR}=2 %.

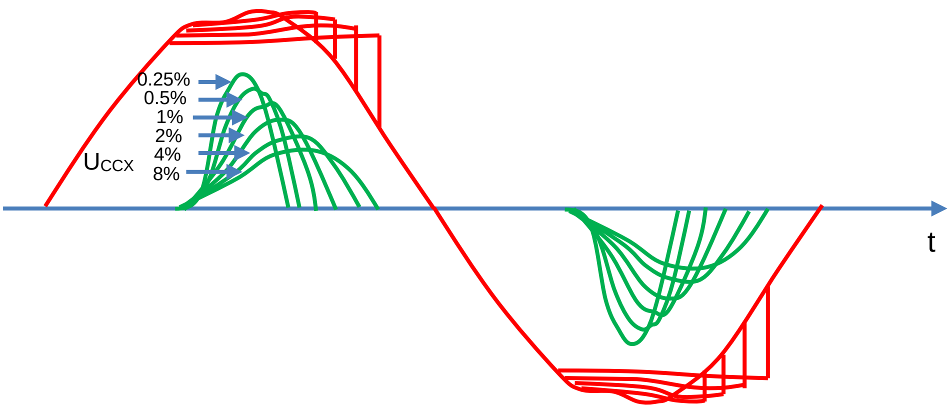


Figure 21. Computer type load current and voltage variation at the input when the short-cirucit voltage UCCX varies from 0.25% to 8% while the short-circuit voltage UCCR stays constant and equal to 2%

Table 1 presents the change of the characteristic parameters for different impedances. When the source impedance increases, the power factor improves whereas the voltage distortion rate in the input of user installation increases.

Table 1. Change of current and voltage characteristic parameters for a computer type load supplied from a constant source of impedance UCCR and equal to 2% for values of UCCX changing from 0.25% to 8%

U_{CCX}	Crest factor	Power factor	Current spectrum H_n%=100 I_N/I_1						Global distortion rate of voltage
%	I_{crest}/I_{rms}	λ	H3	H5	H7	H9	H11	H13	
0.25	2.7	0.64	87	64	38	15	1	7	2.8
0.5	2.63	0.65	85	60	33	11	4	7	3.5
1	2.51	0.68	81	52	24	6	7	6	5.4

2	2.35	0.72	76	42	14	7	6	3	7.5
4	2.19	0.75	69	29	8	8	4	4	11.2
6	2.1	0.77	63	21	8	6	3	3	14.2
8	2	0.78	59	17	8	5	3	2	16.8

Distortion rate value determines the selection of the source. Typically, a distortion rate of 5 % is the limiting value admissible for receiver installations that can be either polluters or polluted.

Curves in Figure 22 show voltage global distortion rate variation in the rectifier's input as a function of two parameters:

- Source short-circuit voltage in 0 to 8 % range; and

- Three values of resistive short-circuit voltage (U_{ccR}=0, U_{ccR}=2% and U_{ccR}=4 %).

They also suggest that inductive short-circuit voltage determines the voltage distortion rate except in situations when the short-circuit voltage is lower than 1 %.

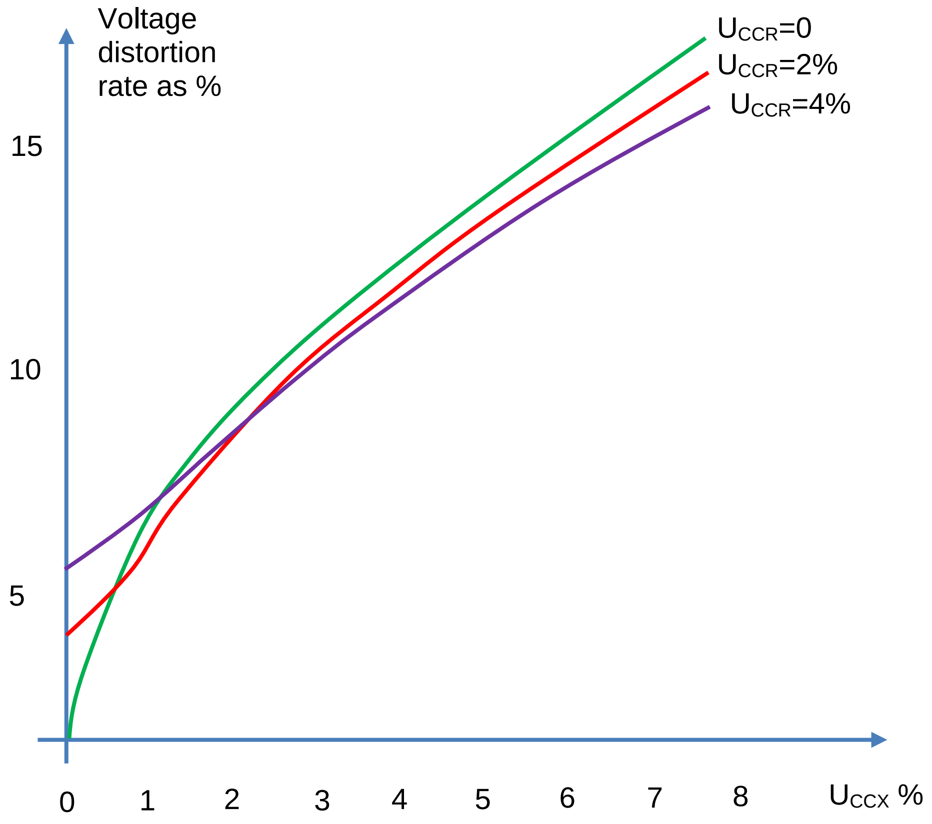


Figure 22. Voltage distortion rate variation at input of microprocessor type load as a function of UCCX and several values of the source's UCCR

Calculation of source power for supplying RCD type loads

If the active power absorbed by the rectifier, Pr, is known, it is important to correctly select the power source, Ps, that must supply it.

A first indication is provided by the power factor:

$$\lambda = \frac{P}{s}$$

Power factor depends on the total short-circuit voltage upstream of rectifier but can be given a mean value of roughly 0.7. With this first criterion, the power of the source must be at least equal to the active power absorbed by the rectifier multiplied by:

$$\frac{1}{0.7}\ \ or\ 1.43$$

The second criterion relates to a distortion rate that would be acceptable:

- If a distortion rate of 5 % is desired, it is possible to keep an inductive 1 % short-circuit voltage (as shown in Figure 22); and
- For a 10 % distortion rate, a 3 % short-circuit voltage must be kept.

For a transformer:

- In the case U_{ccx} = 4 %

- For D = 5 % a power ratio of:

$$\frac{P_s}{P_r} = \frac{4\%}{1\%} = 4\ is\ sufficient$$

For D=10% the power ratio would be:

$$\frac{P_s}{P_r} = \frac{4\%}{3\%} = 1.33$$

However in this case, a value at least equal to 1.43 would be required by the power factor.

- If U_{CCX}=6%
- For D=5% a power ratio of:

$$\frac{P_s}{P_r} = \frac{6\%}{1\%} = 6\ is\ necessary$$

- For D=10%, a power ratio of 2 is needed.

For a transformer, it is typically mandatory to take a much higher power ratio considering that distortions can already exist in the network. A 3 % distortion rate caused solely by the rectifiers, leads one to keep a 0.45 % inductive short-circuit voltage (in line with Figure 22). That amounts to multiplying the transformer power ratings by 2.2 to get a distortion rate of 5 %.

For an alternator:

Since 5 % and 10 % distortion rates lead to inductive short-circuit voltages of 1 % and 3 % respectively, alternator to rectifier power ratios are equal to:

$$\frac{U_{ccx}}{1\%} \quad and \quad \frac{U_{ccx}}{3\%}$$

If U_{ccx}=18%, it will be necessary:

- For D=5% to have a power ratio of:

$$\frac{P_s}{P_r} = 18$$

- For D=10% to have a power ratio of:

$$\frac{P_s}{P_r} = \frac{18\%}{3\%} = 6$$

For an inverter:

Classic single phase inverter exhibits impedance comparable to the impedance of the alternator (with U_{ccx} of the order of 12 %). Since the output distortion of an inverter must be limited to 5 %, it is desirable to keep a power ratio of the order of 12. Classic type inverters are available mostly in three-phase versions. Assuming a 5 % distortion rate, the power ratio is 7 when operated with a transformer whose secondary is ZIGZAG connected.

PWM inverter with adequate regulation (its impedance is at least five times lower than the impedance of a transformer for which the power rating must be multiplied by 4).

As long as the current taken by the load exhibits a crest value lower than the limiting threshold value for the equipment, the distortion rate stays very low and inferior to 5 %. As soon as the threshold limit is reached, the voltage provided by the inverter becomes distorted (sine wave becomes affected by crest flattening) and the voltage distortion rate increases. In order to avoid a voltage distortion surpassing 5 %, it is mandatory to set the current threshold limit at 1.5 times the crest value of the nominal effective current of the inverter.

Therefore, $I_{limit} = 1.5\sqrt{2}\ I_{RMS}$

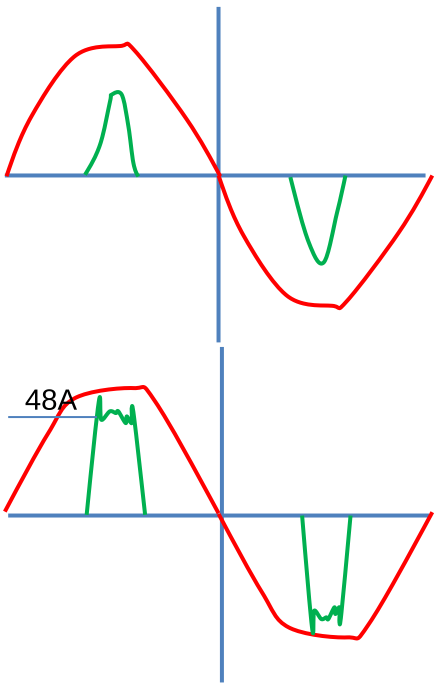
The corresponding current crest factor is then equal to 1.5√2 or 2.12.

Figure 23 presents voltage and current variation in a 5.2 kVA inverter with a threshold limit set at:

$$\frac{5000}{220} \cdot 1.5 \cdot \sqrt{2} = 48\ A$$

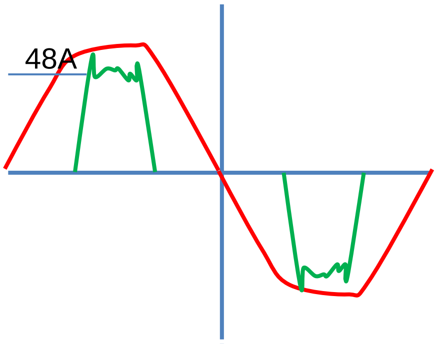
A 5 % voltage distortion rate is reached for an apparent power of 5.2 kVA. That is slightly higher than 5 kVA which is its rated design parameter. The power factor of the RCD load is very close to 0.8 (0.79) and, consequently, the inverter does not need to be over-dimensioned to supply this type of load.

In the example presented in Figure 23, a 5 kVA inverter can supply a 4 kW rectifier with a distortion rate lower than 5 %.

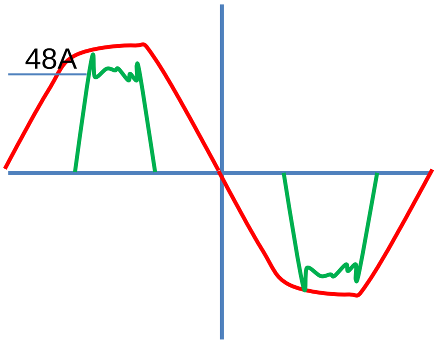


U_{rms}	220 V
I_{rms}	11 A
Power factor	0.61
Crest factor	3.6
Distortion rate	2.7%
Apparent power	2.4 kVA
Active power	1.5 kW

U_{rms}	220 V
I_{rms}	20 A
Power factor	0.69
Crest factor	2.4
Distortion rate	3%
Apparent power	4.4 kVA
Active power	3 kW



U_{rms}	220 V
I_{rms}	24 A
Power factor	0.79
Crest factor	2
Distortion rate	5%
Apparent power	5.2 kVA
Active power	4.17 kW



U_{rms}	220 V
I_{rms}	29 A
Power factor	0.82
Crest factor	1.64
Distortion rate	10%
Apparent power	6.3 kVA
Active power	5.2 kW

Figure 23. Output voltage variation of 5 kVA inverter with threshold limit set at 48 A

Therefore, inverter = $\frac{P_R}{0.8} = 1.25\ P_R$

Limiting the current improves the power factor. The PWM inverter appears to be the ideal voltage source for supplying not only RCD type loads but also all receiver equipment which are generators of harmonic currents (non-linear loads). In the previous section, inverters and single phase loads were described. The same reasoning can be applied to three phase equipment in the case the equipment is equipped with independent regulation in each phase (which is typically the case).

Therefore, static inverters equipped with PWM are almost perfect voltage sources. Besides their qualities in terms of voltage and frequency stability, they are the best generators for supplying electronic and micro-processor loads. The high speed response of their regulation provides very low harmonic impedance. This allows them to provide a low distortion voltage to receivers that are generators of harmonic currents (non-linear loads).

Effects of line impedances on voltage distortions

It was already mentioned that it is suggested to supply loads that are generators of harmonic currents with special lines. This is true for RCD type loads, but also for all loads that use power electronics such as rectifiers, battery chargers, speed controllers etc. The use of a special line isolates harmonics through impedance, as shown in Figure 24.

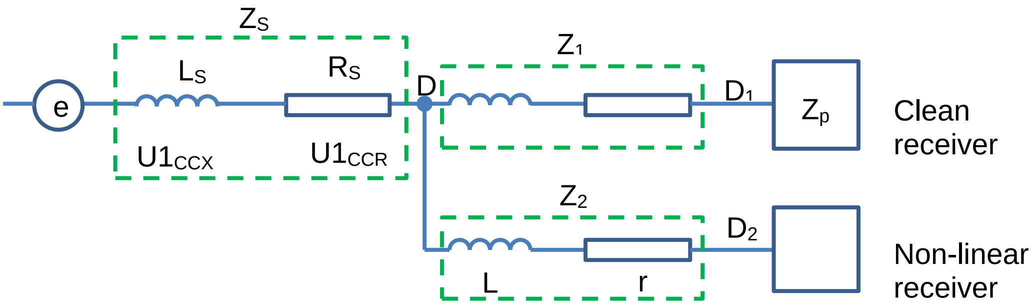


Figure 24. Power supply through a specific line

For a clean load

The distortion rate D_1 is identical to D, and this is more correct as the line impedance Z_1 is small in comparison to the receiver's impedance Z_p.

For a non-linear load

D_2 will be lower as the sum Z_2+Z_s remains low. In other words, the non-linear load has a low power rating in comparison to its supply.

The following example shows the effect of Z_2 on D and D_2. Let's consider a set of micro-computers using 10 kW at 230 V that are supplied by a cable conductor 100 m long, which is connected to a transformer.

- Cable characteristics:
- Section: 10 mm2,
- Lω = 0.1 Ω/km at 50 Hz,
- r = 20 Ω/km for a 1 mm2 section.
- Transformer characteristics:
- 50 kVA (with U_{ccx} = 4% et U_{ccR} = 2 %).

It is mandatory to determine the impedances of the inductive short-circuit and resistive short-circuit of the transformer but referred to the active power of micro-computers. Therefore:

$$U'1ccx = U1ccx\frac{P_R}{P_s}$$

$$U'1ccr = U1ccr\frac{P_R}{P_s}$$

Hence:

$$U'1ccx = 4\% \cdot \frac{10}{50} = 0.8\%$$

$$U'1ccr = 2\% \cdot \frac{10}{50} = 0.4\%$$

-Assuming that Z_2 = 0 (load very close to transformer). Curves in Figure 22 will give D = 4.6 % = D_2.

-It is mandatory to calculate D and D_2 with a line 100 m/10 mm2 (i.e. 100 m long and a section 10mm2):

-Therefore short-circuit impedances of the line referred to P_R:

$$U'2ccx = I\omega\frac{P_R}{U_n^2} \cdot 100$$

$$U'2ccr = R \cdot \frac{P_R}{U_n^2} \cdot 10$$

Hence with:

$$I\omega = 0.1 \cdot \frac{100}{1000} = 10m\Omega$$

$$r = 20 \cdot \frac{100}{1000} \cdot \frac{1}{10} = 0.2\Omega$$

$$U'2ccx = 10 \cdot 10^{-3} \cdot \frac{10^4}{(230)^2} \cdot 100 = 0.19\%$$

$$U'2ccr = 0.2 \cdot \frac{10^4}{(230)^2} \cdot 100 = 3.8\%$$

Total short-circuit impedances:

$$U'ccx = 0.8\% + 0.19\% = 0.99\%$$

$$U'ccr = 0.4\% + 3.8\% = 4.2\%$$

Therefore:

$$U'ccx = U'1ccx + U'2ccx$$

$$U'ccr = U'1ccr + U'2ccr$$

-Voltage distortion rates D'_L and D'_R are referred to as impedances of inductive short-circuits and resistive short-circuits. These values are determined from curves in Figure 25 (a) and Figure 26 (b) and are:

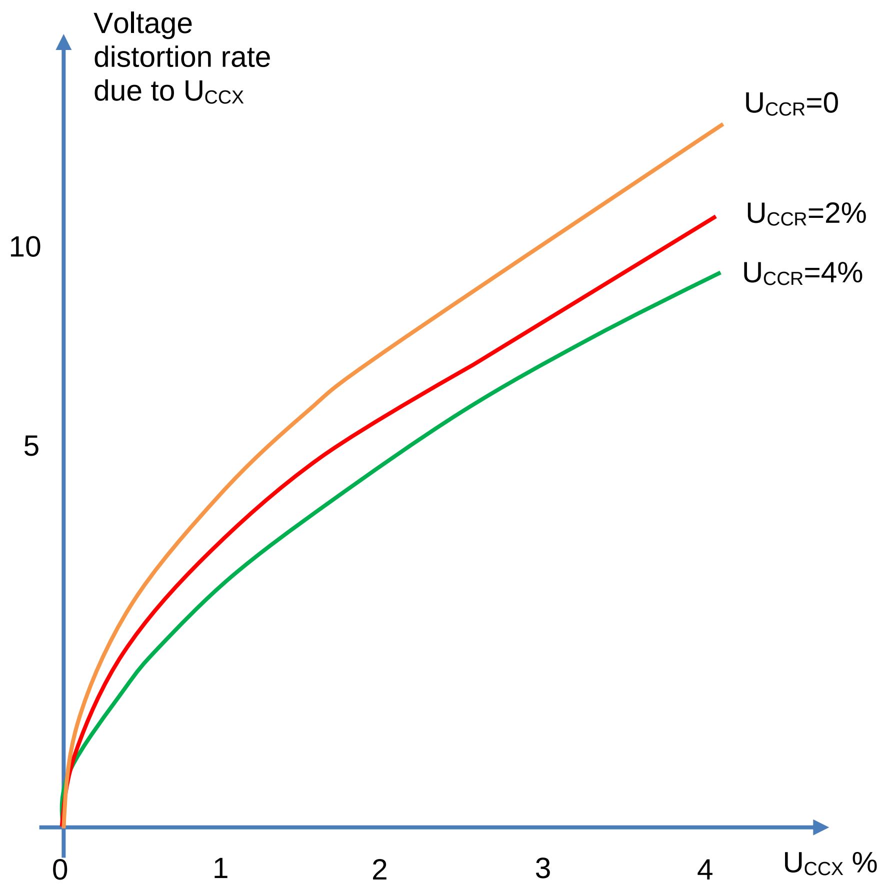


Figure 25. Voltage distortion rates due to UCCX for different values of UCCR

D'_L = 3.9 %; and

D’R = 3.9 %.

-Distortion rate at input of computer equipment:

$$D_2 = \sqrt{(3.9\%)^2 + (3.9\%)^2} = 5.52\ \%$$

-Voltage distortion rates DL and DR at the source:

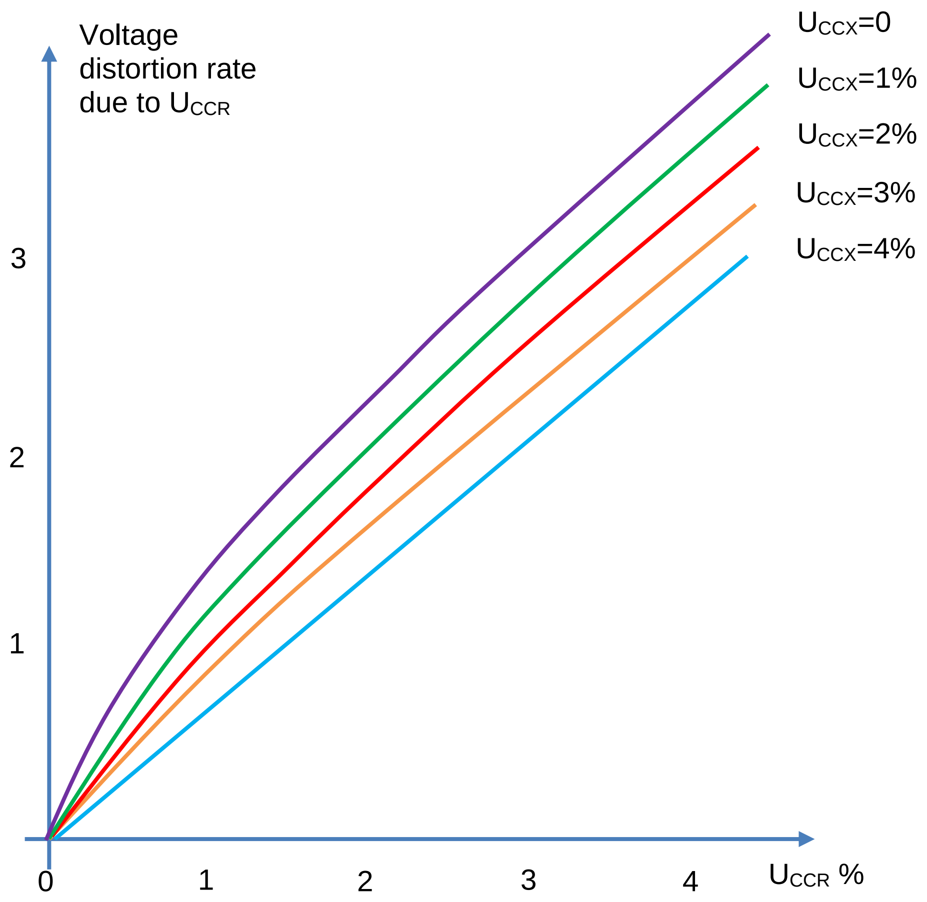


Figure 26. Distortion rates due to UCCR for different values of UCCX

$$D_L = D'_L \frac{U'_{1ccx}}{U'_{ccx}}$$

$$D_R = D'_R \frac{U'_{1ccr}}{U'_{ccr}}$$

Therefore:

$$D_L = 3.9\% \cdot \frac{0.8}{0.99} = 3.15\%$$

$$D_R = 3.9\% \cdot \frac{0.4}{4.2} = 0.37\%$$

-Voltage distortion rate D at the source:

$$D = \sqrt{D_L^2 + D_R^2}$$

$$D = \sqrt{(3.15\%)^2 + (0.37\%)^2} = 3.17\%$$

-In this example, the supply line causes D and D_2 to change as follows:

D from 4.6 % to 3.17 %; and

D2 from 4.6 % to 5.52 %.

Input filters in computer/micro-processor devices

Their purpose is to prevent the propagation of disturbances created by switched mode power supplies towards other equipment installations that could be seriously affected. They help attenuate some network disturbances which are likely to change the functioning of electronic and data information equipment. The question is to know if these filters attenuate harmonic currents generated by RCD loads.

Interference rejection in network

Switched mode power supplies work at high frequencies in order to decrease the size and weight of transformers. In Figure 26, the load resistance is replaced by a transformer and its load. In this circuit, the line current stays identical due to capacitor C.

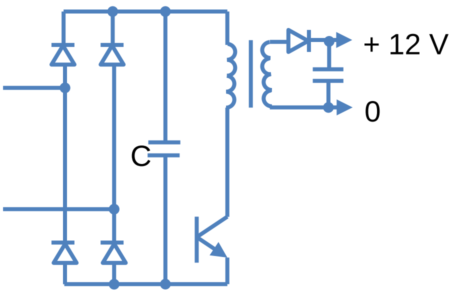


Figure 27. Switched mode power supply to RCD load - basic circuit diagram

To achieve silent operation, the switching frequency is always high and in any case in excess of 20 kHz. Transistor commutation times (change from non-conducting to conducting status and vice versa) are very quick and do not exceed a few tens of nanoseconds. High frequency commutations (switching) produce HF interference that is propagated by conduction and radiation. This increases the presence of parasitic interference along the line upstream of the switching device. In order to limit HF currents circulation, designers of data information processing equipment install filters upstream of the switched mode power supply element. Typical filter circuit is presented in Figure 27.

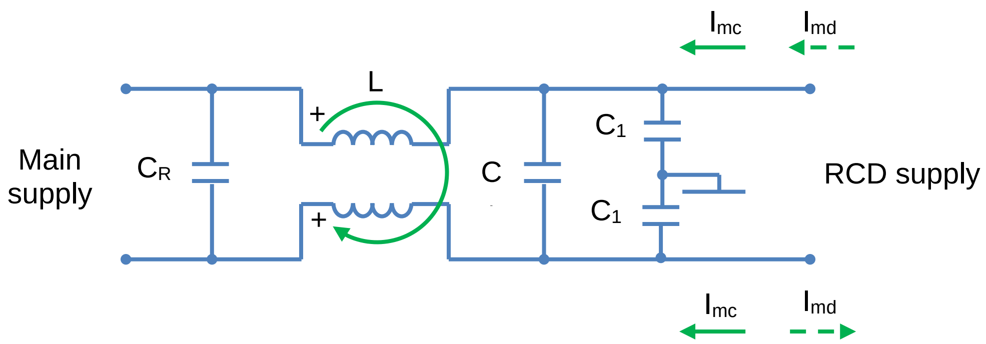


Figure 28. Anti-parasitic interference filter - basic circuit diagram

These filters decrease disturbances of:

-Common mode which affect conductors with respect to ground; and

-Differential mode which exists between the two conductors.

Inductance, L, provides high impedance to currents of common mode but practically none to those of differential mode as its windings are wound in opposition. Disturbances of common mode are conducted to ground by capacitors, C_1, and blocked by inductance, L. Disturbances of differential mode are attenuated by capacitors C_A and C_R which, at high frequency, provide low impedance between the conductors.

Switched mode power supply protection

The filter inserted between the AC mains and the RCD supply provides a second function: it protects the RCD supply from impulse type over-voltages and from HF interference of differential and common mode which exist in the mains.

Leakages to ground

The existence of capacitors C_1 causes a leakage current at 60 Hz to flow to ground. Typically, design standards specify leakage current values that should not be surpassed (a few milli-amperes for equipment connected to a mains point). For example, leakage currents should be kept below 3.5 mA for equipment connected to mains point.

If a line supplies several electronic and data processing devices, the sum of the leakage currents can trip the highly sensitive differential residual current device (30 mA) which is normally inserted in the line.

Harmonics filtering

The filters inserted between the mains and the RCD supply work efficiently in the frequency band-pass that ranges from 10 kHz to 100 MHz. However, they are of no use against harmonic currents injected into the mains network. This is due to the fact that harmonic currents generated by RCD supplies are of relatively low frequency: 1.2 kHz corresponds to a harmonic of order 20 in relation to a fundamental at 60 Hz.

References

- Cahier Technique Merlin Gerin n° 159
- IEC 146-1-1 Semi-conductor converters. General requirements and line commutated convectors - part 1-1: specifications basic requirements
- IEC 950 Safety of information technology equipment including electrical business equipment